THE MULTIPLE CRITERIA OPTIMIZATION MODEL FOR THE REPUBLIC OF MOLDOVA

MODELUL DE OPTIMIZARE CU CRITERII MULTIPLE PENTRU REPUBLICA MOLDOVA

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Abstract

The main goal of this article is to present optimization model for Moldova with multiple criteria, based on the input-output and computable general equilibrium approaches. Here demand and supply models are studied as integrated one composite input-output model. Such presentation carries on the fact that input-output models have dual presentation in demand and supply models. Based on this principle, computable general equilibrium model was examined. Simultaneously, balanced prices between demand and supply were determined. Multiple criteria non-linear programming is applied to balance between output and as well as the prices of demand and supply. Equilibrium is obtained as optimal solution among multiple criteria non-linear programming solutions.

Key words: *input-output model, calculated general equilibrium model, demand and supply models, multiple non-linear criteria, balanced prices, optimal solution.*

JEL : C 61, C68.

1. Introduction

For a long time in the Institute of Mathematics and Computer Sciences "Vladimir Andrunachievici" there were examined: static input-output model [7-10], dynamic input-output model restricted by limited energy resources [11], computable general equilibrium model [12], and the Markov chain approach based on the input-output tables [13]. All these models have been examined using statistic data referring to the input-output table, constructed on the base of 19 and 16 aggregated branches of the national economy [14]. Static and dynamic optimization models were formulated, simulation calculation was done and analysed. Input-output table balancing problem was solved using RAS method [15]. For dynamic model matrix of the investment coefficients was constructed. The emphasis was put on the problem of applying the theory of Markov chain for examination of the 19 and 16 branches in the framework of the input-output model for Republic of Moldova. A square exchange matrix of order $n \times n$ has been constructed. Every branch was considered as one state of the Markov chain with n states. We introduced a new $(n+1)$ absorption state so that the examined matrix became of the order $(n+1)\times(n+1)$. The obtained transition matrix – probabilities matrix has been used for forecasting.

The main goal of this article is referred to the construction of nonlinear optimization model, which combines two approaches [4]. That of the input-output modeling and the calculating general equilibrium approach. This approach was adapted to the Republic of Moldova economy. As a rule, calculated general equilibrium model contains supply and demand models, price models and general equilibrium model, while input-output model in most cases examined only supply side for both aspects. To surpass this, the demand model from the input-output approach was examined, in such a way composite calculated general equilibrium model

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was obtained. Multiple non-linear programming criteria were coming to balance both output and prices.

2. Degree of scientific approach, and it reflections in the specialty publications

Input output models and calculated general equilibrium models are largely studied and utilized in economy activity modeling both abroad and in our country [1-6]. In V. Leontief [5] input-output model is presented as a general equilibrium system. Till now, input-output in the framework of the general equilibrium model was limited to the supply side, so input-output approaches do not describe as a whole general equilibrium system [2], while a general equilibrium theory [1] examined both quantities and prices simultaneously. In [4] unified approaches was proposed.

3. Data sources and methods utilized

Data presented by National Bureau of Statistics, Republic of Moldova [14], Statistic Bank, compartments: Social and Economic Statistics were used for enounced problem solution. And formulated nonlinear optimization problem has been solved using method of the Lagrange multipliers and Solver environment from Excel

4. Problem wording and obtained results.

Now it will be continued with exposure of the unified model keeping in mind both input output aspect and calculated general equilibrium aspect. Model structure in money terms is determined in the following mode.

The model of supply is supposed to input output side of the model

$$X^S = (I - A)^{-1} Y^D \quad (1)$$

Here, X^S - is the vector of total gross output which use disposable primary recourses, Y^D is the final consumption vector, and A - is the direct expenditure coefficients matrix. By means of this model can be determined total gross production of supply final uses being given.

The model of demand also adequate to input output side of the model

$$X^D = N^S (I - R)^{-1} \quad (2)$$

Here, X^D - is the total gross production vector which required primary resource in order to confront primary uses, N^S is the primary input vector, and R - is the direct distribution coefficients matrix.

Direct material expenditure coefficients definition:

$$A_{ij} = X_{ij} / X_j, \; j = 1,...,n; i = 1,...,n \quad (3)$$

Direct material distribution coefficients definition:

$$R_{ij} = X_{ij} / X_i, \; i = 1,...,n; \quad j = 1,...,n. \quad (4)$$

Here, X_{ij} - is the square $(n \times n)$ matrix of the inter branch consumption flows. Every element of this matrix represents output of the branch i earmarked to the branch j, thus being input of the branch j to branch i, X_i - is the line vector $(1 \times n)$ of the gross total production, X_j - is the column vector of the gross total production $(n \times 1)$.

The model (4) is more used in theory than in practice. Using this model one can determine gross output of demand when the primary input is given, and in consequence demand modified itself to supply.

The model of supply price:

$$P^S = A_N (I - A)^{-1} \quad (5)$$

$$A_N = A_g + A_v + A_M$$

Here, P^S - is the supply price, and A_N - is the value added in relation with production, constitute from the A_g - is the direct coefficients of fixed assets depreciation, A_v - is the direct input coefficients of labor income and welfare, - is the A_M direct input coefficients of social profit and taxes.

The model of demand price:

$$P^D = (I - R)^{-1} R_Y \quad (6)$$

$$R_Y = R_z + R_w + R_k + R_f$$

Where, P^D - is the demand price, and R_Y - is the final distribution coefficients, R_z direct distribution coefficients of the fixed capital formation, R_w - is the direct distribution coefficients of consumption, R_k - is the direct distribution coefficients of accumulation, and R_f - is the direct distribution coefficients of export.

General equilibrium model:

$$X^S = X^D \quad (7)$$

$$P^S = P^D \quad (8)$$

Following conditions must be satisfied in order to get general equilibrium

$$(P^S)^T = P^D$$

$$(P^S)^T = [(I - A)^{-1}] \cdot A_N^T$$

$$P^D = (I - R)^{-1} \cdot R_Y \quad (9)$$

$$A_N = R_Y^T \cdot H$$

$$H = [(I - R)^{-1}]^T \cdot (I - A)$$

Where, P^S - is the supply price, P^D - is the demand price, A - is the direct expenditure coefficients matrix, R - is the direct distribution coefficients matrix, A_N - is the value added in relation with production, R_Y - is the final distribution coefficients.

Input output general equilibrium model with multiple non-linear criteria

$\max : L_S = f(X^S, Y^D)$ - revenue function from supply side

$\max : L_D = f(X^D, N^S)$ - total utility function from demand side

$$\begin{cases} X^S - (I - A)^{-1} Y^D \le 0 \\ X^D - N^S (I - R)^{-1} \ge 0 \\ BX^S \le \overline{B} \\ F(X^D, N^S) \ge 0 \\ X^S = X^D \\ X^D \ge 0,\ N^S \ge 0 \\ X^D \ge 0, Y^D \ge 0. \end{cases} \quad (10)$$

Here, B- is the matrix of the resource consumption coefficients, $\overline{B}$- is the gross resources available vector, $F(X^D, N^S) \geq 0$- is function dependence on vector X^D total gross output, and N^S primary inputs vector, rest of the variables being defined eerily. $\max : L = \alpha_1 \cdot f_S(X^S, Y^D) + \alpha_2 \cdot f_D(X^D, N^S)$

$$\begin{cases} X^S - (I - A)^{-1} Y^D \leq 0 \\ X^D - N^S (I - R)^{-1} \geq 0 \\ BX^S \leq \overline{B} \\ F(X^D, N^S) \geq 0 \\ X^S = X^D \\ X^D \geq 0,\ N^S \geq 0 \\ X^D \geq 0, Y^D \geq 0. \end{cases} \tag{10}$$

As a production $f_1(X^S, Y^D)$ and utility $f_2(X^D, N^S)$ activities were considered constant elasticity of substitution functions (CES) which have the following forms that are based on the observed quantities, prices and budget shares from the data: $y = f(x) = \bar{y} \cdot \left(\sum_{i=1}^{n} \theta_i (x_i / \bar{x}_i)^{\rho} \right)^{1/\rho}$, where $x_1, x_2, ..., x_n$ denotes inputs and the elasticity of substitution is given by $\sigma = 1/(1-\rho)$, and $\theta_l = \overline{p}_i \overline{x}_i / \sum_{j=1}^{n} \overline{p}_j \overline{x}_j ; i \neq j$, denote benchmark value share of input i in p_i total production value given input prices. Benchmark quantities and prices are denoted with bar. Going from this form, two functional forms for production and utility functions have been calibrated in the following mode:

$$f_1(X^S, Y^D) = Y_0^D \cdot (\sum_{i=1}^{n} \theta_i \cdot (X_i^S / X_{0i}^S)^{\rho})^{1/\rho}$$

$$f_2(X^D, N^S) = N_0^S \cdot (\sum_{i=1}^{n} \eta_i \cdot (X_i^D / X_{0i}^D)^{\rho})^{1/\rho}$$

$$\theta_i = X_{0i}^S / \sum_j X_{0j}^S, \quad \eta_i = X_{0i}^D / \sum_j X_{0j}^D .$$

	θ	η	Y_0^D	X_0^S	N_0^S	X_0^D	$\overline{B}$	AX
1	0,1252	0,0847	27733459	14370988	27733459	14370988	22105708	17523741
2	0,0004	0,0003	92610	49839	92610	49839	70757	55064
3	0,0046	0,0031	1025889	483969	1025889	483969	896506	618895
4	0,2393	0,1619	53015535	13009693	53015535	13009693	66182180	53261945
5	0,0303	0,0205	6722969	2132638	6722969	2132638	7593842	6863392
6	0,0865	0,0585	19167283	3780068	19167283	3780068	25455268	29191510
7	0,1270	0,0859	28124561	15122584	28124561	15122584	21509338	19832542
8	0,0136	0,0092	3022326	1275618	3022326	1275618	2889602	2527549
9	0,0831	0,0563	18416766	5338993	18416766	5338993	21634729	18267358
10	0,0371	0,0251	8215642	5231762	8215642	5231762	4936271	3991930
11	0,0307	0,0208	6804951	5019318	6804951	5019318	2953994	2438788
12	0,0474	0,0321	10502540	5797956	10502540	5797956	7782854	9229113
13	0,0006	0,0004	128996	78290	128996	78290	83884	73128
14	0,0114	0,0077	2516077	1292276	2516077	1292276	2024551	1818369
15	0,0026	0,0018	586650	381193	586650	381193	339890	1482644
16	0,0246	0,0167	5454490	2387134	5454490	2387134	5074367	5047590
17	0,0313	0,0212	6934921	4621462	6934921	4621462	3827186	3352238

18	0,0410	0,0277	9070672	6333150	9070672	6333150	4528718	3822472
19	0,0342	0,0232	7583801	4663414	7583801	4663414	4831234	4327767
20	0,0047	0,0032	1037186	701275	1037186	701275	555702	681907
21	0,0085	0,0058	1889163	843253	1889163	843253	1730262	875596
22	0,0114	0,0077	2518677	1116034	2518677	1116034	2320411	1696259
23	0,0042	0,0029	939064	472652	939064	472652	771591	1641994

Sources: Elaborated by the author

As a weighting values were selected $\alpha_1 = \alpha_2 = 0{,}5$, as $\sigma = 1/(1-\rho), \rho = (\sigma - 1)/\sigma$, and, if $\sigma = 0{,}5,\ \rho = -1$. Vectors θ and η values were calibrated proceeded from base year data of the year 2014, $\theta_l, \eta_i; i = 1,...,23$.

It will be mentioned that direct material expenditure coefficients, direct distribution coefficients, gross resource available coefficients matrixes A, R, B and two inverse matrixes $(I-A)^{-1}, (I-R)^{-1}$ were constructed using available material and resource flows data.

Further some initial and calibrated data follows:

General equilibrium model unified with input output model $\max : 0{,}5 \cdot Y_0^D \cdot (\sum_{i=1}^{n} \theta_i \cdot (X_i^S / X_{0i}^S)^\rho)^{1/\rho} + 0{,}5 \cdot N_0^S \cdot (\sum_{i=1}^{n} \eta_i \cdot (X_i^D / X_{0i}^D)^\rho)^{1/\rho}$

$$\begin{cases} X^S - (I-A)^{-1} Y^D \le 0 \\ X^D - N^S (I-R)^{-1} \ge 0 \\ BX^S \le \overline{B} \\ F(X^D, N^S) \ge 0 \\ X^S = X^D \\ X^D \ge 0,\ N^S \ge 0 \\ X^D \ge 0, Y^D \ge 0. \end{cases} \tag{11}$$

Matrix: A, R, B were constructed, Functional form coefficients $\theta_l, \eta_i; i = 1,...,23$ referred to production and utility functions were calibrated. Base year initial data about unknown variables: X^S, Y^D, X^D, N^S and recourse restriction vector, $\overline{B}$ were defined. Further, it must be necessary to apply SOLVER tool for optimization problem solving. With a view to this objective all Solver parameters have been determined and introduced. It is referred to objective function functional form; to matrixes $A, R, (I-A)^{-1}, (I-R)^{-1}, B$; all restrictions defined by the model such as $(R * X^D - X^D - N^S) \ge 0,\ X^S X^D,\ X^D \ge 0,\ N^S \ge 0,\ X^S \ge 0, Y^D \ge 0$ are satisfied; base year 2014 data was balanced; components of the objective functions: revenue function from supply side and total utility function from demand side were calibrated.

Father, calculation results for years 2019, 2020 will be presented. Objective function takes following values for these years: year 2019 – *306238319* and year 2020 – *309300702*.

Base year, 2014 data				*Calculation results, year 2019*			
XS0	*YD0 (VAD)*	*XD0*	*NS0*	*XS*	*YD=VAD*	*XD*	*NS*
27733459	14370988	27733459	14370988	38450952	20927211	38450952	20927211
92610,47	49839,36	92610,47	49839,36	124910	69846,02	124910	69846,02
1025889	483968,6	1025889	483968,6	1159300	540405	1159300	540405
53015535	13009693	53015535	13009693	76970362	23708418	76970362	23708418
6722969	2132638	6722969	2132638	11568423	4705031	11568423	4705031
19167283	3780068	19167283	3780068	47665014	18473504	47665014	18473504
28124561	15122584	28124561	15122584	47826737	27994195	47826737	27994195

3022326	1275618	3022326	1275618	4875582	2348033	4875582	2348033
18416766	5338993	18416766	5338993	28712212	10444854	28712212	10444854
8215642	5231762	8215642	5231762	11582331	7590401	11582331	7590401
6804951	5019318	6804951	5019318	9742978	7304190	9742978	7304190
10502540	5797956	10502540	5797956	24520246	15291133	24520246	15291133
128996	78289,8	128996	78289,8	201354,4	128226,2	201354,4	128226,2
2516077	1292276	2516077	1292276	4148962	2330593	4148962	2330593
586650,1	381193,2	586650	381193,2	5537947	4055302	5537947	4055302
5454490	2387134	5454490	2387134	10487298	5439708	10487298	5439708
6934921	4621462	6934921	4621462	10789420	7437182	10789420	7437182
9070672	6333150	9070672	6333150	13410884	9588413	13410884	9588413
7583801	4663414	7583801	4663414	12245281	7917515	12245281	7917515
1037186	701275	1037186	701275	2418673	1736766	2418673	1736766
1889163	843253	1889163	843253	1228093	352497,4	1228093	352497,4
2518677	1116034	2518677	1116034	3108518	1412259	3108518	1412259
939064	472652	939064	472652	4441212	2799218	4441212	2799218

Sources: Elaborated by the author

Calculation results, year 2020			
XS	*YD=VAD*	*XD*	*NS*
38835462	21136483	38835462	21136483
126159	70544	126159	70544
1170893	545809	1170893	545809
77740066	23945502	77740066	23945502
11684107	4752081	11684107	4752081
48141664	18658239	48141664	18658239
48305005	28274137	48305005	28274137
4924338	2371513	4924338	2371513
28999334	10549303	28999334	10549303
11698154	7666305	11698154	7666305
9840408	7377232	9840408	7377232
24765448	15444044	24765448	15444044
203368	129509	203368	129509
4190452	2353899	4190452	2353899
5593326	4095855	5593326	4095855
10592171	5494105	10592171	5494105
10897314	7511554	10897314	7511554
13544993	9684297	13544993	9684297
12367734	7996690	12367734	7996690
2442860	1754133	2442860	1754133
1240374	356022	1240374	356022
3139603	1426381	3139603	1426381
4485624	2827210	4485624	2827210

Sources: Elaborated by the author

Conclusions

Problem examined in these notes is referred to determining optimal solution of the non linear multi-criteria programming model, which combined two approaches. One of them representing input output approach and the other dealing with computable general equilibrium approach. Using stat data for Republic of Moldova about principal macroeconomic indicators, Social Accounting Matrix for 2014 year was constructed. Both revenue function from supply side and total utility function from demand side were calibrated based on the stat data about inputs and outputs: X^S, Y^D, X^D, N^S. Also direct input coefficients matrix, direct distribution coefficients matrix, resource consuming coefficients matrix: A, B, R for years 2019, 2020 have been calculated starting from the 2014 matrix structure of the material flows and resource components (8.Annexes). Coming with unified objective function, based on two calibrated behavioral functions, optimization problem was formulated supposed to all unknown variables restrictions from demand and supply side, equilibrium conditions, non negative restrictions over all unknown variables. Obtained optimization problem was solved using Solver tool. Received solution demonstrates good approximation with respect to known values of the seeking variables.

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8. Annexes

Annex 1

A - matrix of the direct material expenditure coefficients

0,177	0,000	0,216	0,140	0,018	0,000	0,000	0,002	0,008	0,005	0,003	0,003	0,000	0,006	0,008	0,007	0,000	0,000	0,000	0,000	0,000	0,000
0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,006	0,004	0,000	0,000	0,000	0,005	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,248	0,762	1,515	0,495	0,144	0,024	0,007	0,025	0,057	0,036	0,035	0,037	0,008	0,012	0,033	0,067	0,000	0,003	0,000	0,054	0,000	0,000
0,000	0,000	1,392	0,039	0,122	0,005	0,001	0,003	0,007	0,017	0,031	0,003	0,000	0,008	0,002	0,014	0,000	0,001	0,000	0,016	0,000	0,000
0,005	0,000	2,312	0,325	0,017	0,018	0,001	0,052	0,016	0,023	0,019	0,040	0,000	0,009	0,001	0,005	0,000	0,001	0,000	0,039	0,000	0,000
0,004	0,082	0,238	0,096	0,102	0,007	0,005	0,097	0,113	0,053	0,192	0,155	0,000	0,000	0,000	0,191	0,000	0,004	0,001	0,060	0,000	0,000
0,011	0,050	0,059	0,017	0,023	0,002	0,000	0,001	0,001	0,010	0,005	0,005	0,006	0,005	0,001	0,009	0,000	0,001	0,000	0,026	0,000	0,000
0,000	0,000	0,070	0,143	0,044	0,011	0,005	0,078	0,164	0,049	0,047	0,039	0,028	0,009	0,002	0,030	0,000	0,003	0,000	0,014	0,000	0,000
0,000	0,000	0,013	0,021	0,066	0,007	0,001	0,004	0,014	0,063	0,006	0,004	0,002	0,010	0,002	0,021	0,000	0,003	0,000	0,007	0,000	0,000
0,000	0,000	0,005	0,007	0,007	0,007	0,001	0,003	0,003	0,014	0,110	0,004	0,001	0,008	0,001	0,017	0,000	0,001	0,000	0,010	0,000	0,000
0,000	0,000	0,300	0,063	0,113	0,013	0,002	0,012	0,007	0,045	0,021	0,053	0,005	0,019	0,000	0,052	0,000	0,001	0,000	0,026	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,002	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,001	0,000	0,000
0,000	0,000	0,009	0,011	0,003	0,000	0,000	0,005	0,004	0,026	0,010	0,007	0,000	0,069	0,004	0,005	0,000	0,000	0,000	0,003	0,000	0,000
0,003	0,000	0,020	0,007	0,008	0,008	0,000	0,006	0,004	0,003	0,002	0,002	0,008	0,007	0,033	0,005	0,000	0,000	0,000	0,005	0,000	0,000
0,000	0,000	0,010	0,039	0,006	0,004	0,003	0,026	0,016	0,040	0,008	0,019	0,002	0,006	0,004	0,037	0,000	0,004	0,000	0,011	0,000	0,000
0,006	0,079	0,004	0,027	0,009	0,004	0,001	0,010	0,007	0,016	0,010	0,016	0,164	0,006	0,003	0,005	0,000	0,000	0,000	0,006	0,000	0,000
0,001	0,025	0,231	0,027	0,050	0,009	0,000	0,016	0,003	0,006	0,003	0,005	0,126	0,009	0,005	0,026	0,000	0,003	0,000	0,009	0,000	0,000
0,001	0,090	0,197	0,038	0,027	0,006	0,000	0,006	0,003	0,006	0,007	0,002	0,259	0,017	0,006	0,005	0,000	0,003	0,020	0,011	0,000	0,000
0,000	0,000	0,021	0,005	0,003	0,001	0,000	0,013	0,001	0,004	0,006	0,001	0,004	0,004	0,000	0,002	0,000	0,001	0,000	0,014	0,000	0,000
0,000	0,000	0,021	0,009	0,009	0,000	0,000	0,004	0,001	0,001	0,001	0,001	0,001	0,000	0,000	0,002	0,000	0,000	0,000	0,002	0,000	0,000
0,002	0,000	0,022	0,013	0,009	0,002	0,000	0,025	0,002	0,008	0,003	0,004	0,128	0,003	0,000	0,007	0,000	0,000	0,000	0,008	0,000	0,000
0,001	0,000	0,097	0,012	0,023	0,002	0,000	0,006	0,002	0,005	0,003	0,006	0,000	0,003	0,000	0,003	0,000	0,000	0,000	0,014	0,000	0,000

Sources: Elaborated by the author

Annex 2

B – matrix of the recourse consuming coefficients expenditure

Sources: Elaborated by the author

0,474	-0,085	0,000	0,022	0,002	0,000	0,002	-0,007	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,455	-0,300	0,000	0,580	0,153	0,000	0,001	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,520	-6,058	1,211	0,157	0,042	0,353	0,088	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,743	-1,276	0,043	0,182	0,025	0,482	0,044	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,672	-0,194	0,002	0,008	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,791	-0,022	0,000	0,013	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,455	0,000	0,006	0,042	0,000	-1,010	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,569	-0,200	0,001	0,019	0,000	0,000	0,000	-0,031	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,699	-0,321	0,000	0,021	0,000	0,000	-0,134	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,357	-0,113	0,000	0,040	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,258	-0,080	0,000	0,008	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,441	-0,015	0,000	0,004	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,387	-0,278	0,001	0,005	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,479	-0,256	0,000	0,007	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,345	0,005	0,001	0,049	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,554	-0,082	0,000	0,003	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,328	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,297	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,379	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,319	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00 0	0,00	0,00	0,00	0,00	0,00	0,00
0,545	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,548	-0,016	0,001	0,000	0,000	0,000	0,000	-0,001	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
0,489	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Sources: Elaborated by the author

Annex 3

R - matrix of the dyrect distribution coefficients

0,177	0,000	0,007	0,272	0,005	0,000	0,000	0,000	0,006	0,001	0,001	0,002	0,000	0,001	0,001	0,002	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,162	0,000	0,001	0,225	0,024	0,000	0,000	0,001	0,006	0,006	0,003	0,009	0,000	0,001	0,000	0,015	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,360	0,031	0,013	0,000	0,002	0,102	0,002	0,002	0,005	0,000	0,000	0,000	0,002	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,128	0,001	0,025	0,495	0,020	0,012	0,004	0,002	0,021	0,006	0,005	0,011	0,000	0,001	0,002	0,008	0,000	0,001	0,000	0,002	0,000	0,000	0,000
0,000	0,000	0,162	0,273	0,122	0,017	0,006	0,001	0,018	0,018	0,028	0,006	0,000	0,003	0,001	0,012	0,000	0,001	0,000	0,003	0,000	0,000	0,000
0,000	0,000	0,045	0,075	0,034	0,005	0,002	0,000	0,005	0,005	0,008	0,002	0,000	0,001	0,000	0,003	0,000	0,000	0,000	0,001	0,000	0,000	0,000
0,004	0,000	0,063	0,534	0,004	0,016	0,001	0,005	0,010	0,006	0,004	0,019	0,000	0,001	0,000	0,001	0,000	0,000	0,000	0,002	0,000	0,000	0,000
0,035	0,002	0,064	1,553	0,234	0,061	0,046	0,097	0,667	0,134	0,408	0,728	0,000	0,000	0,000	0,393	0,000	0,013	0,002	0,029	0,000	0,000	0,000
0,016	0,000	0,003	0,048	0,009	0,003	0,001	0,000	0,001	0,004	0,002	0,004	0,000	0,001	0,000	0,003	0,000	0,000	0,000	0,002	0,000	0,000	0,000
0,000	0,000	0,007	0,921	0,040	0,036	0,021	0,031	0,383	0,049	0,040	0,073	0,000	0,003	0,001	0,025	0,000	0,003	0,000	0,003	0,000	0,000	0,000
0,000	0,000	0,002	0,158	0,071	0,026	0,004	0,002	0,038	0,075	0,006	0,009	0,000	0,004	0,001	0,020	0,000	0,004	0,000	0,002	0,000	0,000	0,000
0,000	0,000	0,000	0,023	0,003	0,011	0,003	0,001	0,004	0,008	0,050	0,004	0,000	0,001	0,000	0,008	0,000	0,001	0,000	0,001	0,000	0,000	0,000
0,000	0,000	1,889	####	6,111	2,578	0,505	0,275	0,911	2,670	1,033	5,832	0,005	0,380	0,000	2,515	0,000	0,046	0,000	0,295	0,000	0,000	0,000
0,000	0,000	0,000	0,007	0,001	0,000	0,000	0,000	0,001	0,000	0,001	0,008	0,000	0,000	0,000	0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,002	0,183	0,008	0,001	0,000	0,005	0,021	0,069	0,021	0,035	0,000	0,061	0,004	0,010	0,000	0,001	0,000	0,002	0,000	0,000	0,000
0,010	0,000	0,003	0,057	0,009	0,032	0,001	0,003	0,010	0,004	0,002	0,005	0,000	0,003	0,016	0,005	0,000	0,000	0,000	0,001	0,000	0,000	0,000
0,000	0,000	0,001	0,274	0,006	0,016	0,013	0,011	0,040	0,044	0,007	0,039	0,000	0,002	0,002	0,033	0,000	0,005	0,000	0,002	0,000	0,000	0,000
0,016	0,001	0,000	0,148	0,007	0,011	0,002	0,003	0,015	0,014	0,007	0,026	0,002	0,002	0,001	0,004	0,000	0,000	0,000	0,001	0,000	0,000	0,000
0,003	0,000	0,024	0,168	0,045	0,030	0,001	0,006	0,006	0,006	0,003	0,010	0,002	0,003	0,002	0,021	0,000	0,004	0,000	0,002	0,000	0,000	0,000
0,025	0,005	0,110	1,283	0,128	0,109	0,009	0,012	0,038	0,031	0,032	0,019	0,023	0,030	0,013	0,021	0,000	0,019	0,107	0,011	0,000	0,000	0,000
0,000	0,000	0,016	0,205	0,019	0,015	0,001	0,035	0,010	0,030	0,036	0,013	0,000	0,009	0,000	0,012	0,000	0,006	0,000	0,019	0,000	0,000	0,000
0,000	0,000	0,008	0,209	0,030	0,005	0,000	0,006	0,005	0,005	0,003	0,006	0,000	0,000	0,000	0,005	0,000	0,000	0,000	0,001	0,000	0,000	0,000
0,024	0,000	0,008	0,287	0,026	0,017	0,001	0,034	0,013	0,025	0,007	0,028	0,007	0,004	0,000	0,019	0,000	0,000	0,000	0,005	0,000	0,000	0,000

Sources: Elaborated by the author